Permutations and Combinations

Permutations

1. 5-digit numbers are to be formed from the digit 1, 2, 3, 4, 5, 6, 7, 8. If no repetitions are allowed, and the first place, third place, and fifth place of the numbers must be odd digits, how many numbers can be formed?

(Ans. 480)

2. If no repetitions are allowed, how many 4-digit numbers greater than 2100 can be formed using the digits 1, 2, 3, 4, 5, 0? (Ans. 228)

3. If no repetitions are allowed, how many 9-digit numbers can be formed from the digits 1, 2, 3, …, 9? How many of these are divisible by 4 ? (Ans. 362880 80640)

4. 5 Chinese books and 3 English books are to be arranged in a row. If the English books must be placed together, how many possible arrangements are there? (Ans. 4320)

5. If repetitions are not allowed, how many 5-digit numbers can be formed from the digits 1, 2, 3, 4, 5 ? What is the sum of the numbers so formed? If repetitions are allowed, what would be the results?

(Ans. 120, 3999960, 3125, 104165625)

6. 8 different books are to be places on a shelf, what is the number of arrangements if two particular books must be separated from each other? (Ans. 30240)

7. In how many ways can 5 gentlemen and 5 ladies sit down at a round table so that no two ladies may be together? (Ans. 2880)

8. In how many ways can the letters of the word FACTORING be arranged,

(a) without changing the order of the vowels a, o, i ;

(b) without changing the order of the consonants f, c, t, r, n, g ;

(c) without changing the order of both the vowels and consonants? (Ans. (a.) 60480 (b.) 504 (c.) 84)

9. 2m white counters and 2n red counters are arranged in a straight line with (m + n) counters on each side of a central mark. Find how many of the arrangements are symmetrical with respect to this mark.

(Ans. $\frac{(m+n)!}{m!n!}$)

10. In how many ways can 2n letters $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n$, be arranged in a line so that the suffixes of the letters x and also those of the letters y are respectively in ascending order of magnitude?

(Ans. $\frac{(2n)!}{n!n!}$)

11. **(i)** Find the number of ways putting p noughts and q crosses on a line, p > q, if no two crosses are together and if a nought comes at each end.

(ii) Find the number of ways in which p like jobs may be assigned to (q + 1) men, if no man is left unemployed, p > q . (Ans. (i), (ii) $\frac{(p-1)!}{(p-q-1)!q!}$)

Combination

1. There are n points in a plane, of which no three are in a straight line, except p, which are all in are straight line. Find the number **(a)** of straight lines formed by using the points **(b)** of triangles formed by them.

(Ans. ${}_nC_2 - {}_pC_2 + 1 \quad {}_nC_3 - {}_pC_3$)

2. How many triangles can be formed by joining any three vertices of a polygon with n sides, the triangles having no sides in common with the polygon?

(Ans. $\frac{1}{6}n(n-4)(n-5)$)

3. How many parallelogram can be formed by the intersections of a set of 10 parallel lines with another set of 7 parallel lines in the same plane?

(Ans. 135)

4. In how many ways can a committee of 3 women and 4 men are chosen form 8 women and 7 men? What is the number of ways if Miss X refuses to serve if Mr. Y is a member?

(Ans. 1960 1540)

5. In how many ways can a committee of 2 Englishmen, 2 Frenchmen, 1 American be chosen form 6 Englishmen, 7 Frenchmen, 3 American? In how many of these ways do a particular Englishman and a particular Frenchman belong to a committee?

(Ans. 945, 90)

6. What is the greatest number of points of intersection made by m straight lines and n circles?

(Ans. $\frac{1}{2}m(m-1)+n(2m+n-1)$)

Miscellaneous

1. Find the number of selections of n things form 2n things of which n are alike and the rest are unlike.

(Ans. 2^n)

2. In how many ways can r things be selected form n unlike things, if two particular things must not occur in the same selection?

(Ans. $\frac{(n+r-1)[(n-2)!]}{r!(n-r-1)!}$)

3. There are 3 pigeon holes marked A, B, C. In how many ways can I arrange 10 different postcards so that 5 of them are in A, 3 in B and 2 in C?

(Ans. 2520)

4. In how many ways can 9 different books be labelled, so that 4 of them have red labels, 3 have blue labels, and 2 have green labels?

(Ans. 1260)

5. Proved that, if n and p are positive integers, $(np)!$ is a multiple of $(p!)^n n!$.

6. Prove that (2n+1) (2n+3) (2n+5) … (4n-3)(4n-1) equals $\left(\frac{1}{2}\right)^n \frac{n!(4n)!}{(2n)!(2n!)}$.